Performance Study of One-Way Absorbing Boundary Equations in 3-D TLM for Dispersive Guiding Structures

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Abstract

Several absorbing boundary conditions based on one-way wave equations (mostly applied to the FD-TD method) have been studied and adapted for the 3-D Symmetrical Condensed Node analysis of guiding structures. The Absorbing Boundary Condition based on perfect absorption of waves at two incident angles has given superior results when compared to that of other absorbing boundaries. Reflections less than 2% over a large frequency spectrum have been obtained for dispersive structures like microstrip and finlines. These excellent absorbing boundary conditions can considerably reduce the computational domain, thus making possible the 3-D TLM analysis of planar and quasi-planar structures with moderate computer resources.

1.0 Introduction

The two major advantages of time domain numerical methods such as Transmission Line Matrix (TLM) method and Finite Difference-Time Domain (FD-TD) method are the ability to analyze complex structures with arbitrary geometry, and the ability to get the frequency domain results over a very wide frequency spectrum with just one time domain simulation. To utilize the latter feature, and to simulate a virtual extension of the limited computational domain, wideband absorbing boundary conditions are a must. These are particularly important when computational resources are limited.

The perfect absorbing boundary conditions are usually global in nature (like the Johns Matrix approach [1]), which makes them quite expensive to implement (especially for structures like microstrip lines and finlines) and requires excessively large computer memory and time. The local absorbing boundary conditions, which make use of only the fields at the neighbouring space and time nodes, are relatively inexpensive to implement, but they will always have some degree of spurious reflections. Many local absorbing boundary conditions for the FD-TD method have been reported, but very few of them have been applied to TLM analysis [2], [3]. The most popular absorbing boundaries are the one proposed by Mur [4] which is based on the work of Engquist and Majda, and the other by Keys and Higdon [5], [6], which allows the propagation direction to be incorporated into the design of absorbing boundary conditions. Factors, such as the finite discretization, dispersion of the mesh and dispersion of the structure affect the quality of the absorbing boundaries. Very recently, Railton and Daniel [7] have studied the effect of finite discretization on several types of absorbing boundaries. In TLM, the implementation of absorbing boundaries is further complicated by the additional dispersion introduced by reactive stubs and the presence of spurious modes supported by the condensed TLM node.

In this paper, we have successfully adapted and implemented several one-way absorbing boundary conditions for Condensed Node TLM analysis of dispersive guiding structures. Reflections less than 2% have been achieved over a wide frequency bandwidth for microstrip and finlines for the first time in TLM Modeling.

2.0 Theory

Consider the wave equation,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}-\frac{1}{v_i^2}\frac{\partial^2}{\partial t^2}\right)E = 0 \qquad (1)$$

where v_i is the velocity of propagation of the waves, which is usually a function of frequency in dispersive guiding structures. Since in most of the guiding structures, the major direction of powerflow is in the propagation direction, the first-order boundary condition in the yz-plane for wave propagation in positive x-direction can be written as,

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$$(\frac{\partial E}{\partial x}+\frac{1}{v_i}\frac{\partial E}{\partial t}) = 0 \qquad (2)$$

The above absorbing boundary operator absorbs well only at the frequency corresponding to the velocity. To absorb better over a moderately wide frequency spectrum, absorbing boundary operators which can perfectly absorb waves traveling in any two directions have been proposed [5], [8]. They can be written as

$$(\frac{\partial E}{\partial x}+\frac{1}{v_1}\frac{\partial E}{\partial t})\,(\frac{\partial E}{\partial x}+\frac{1}{v_2}\frac{\partial E}{\partial t}) = 0 \qquad (3)$$

In numerical form, the above equation for the field value E on the last node n_x(in the x-direction) and k[th] time iteration can be written as

$$E^k(n_x, j, k) = 2\,E^{k-1}(n_x\text{-}1, j, k) - E^{k-2}(n_x\text{-}2, j, k) + (E^k(n_x\text{-}1, j, k) - E^{k-2}(n_x\text{-}1, j, k) + E^{k-1}(n_x\text{-}2, j, k))\,(\gamma_1+\gamma_2) - (E^{k-2}(n_x, j, k) - 2\,E^{k-1}(n_x\text{-}1, j, k) + E^k(n_x\text{-}2, j, k))\gamma_1\gamma_2 \qquad (4)$$

where $\gamma_i = \frac{1-\rho_i}{1+\rho_i}$, $\rho_i = \frac{v_i}{\Delta x}\Delta t = \frac{1}{2\sqrt{\varepsilon_{eff}(f_i)}}$

$\varepsilon_{eff}(f_i)$ is the frequency dependent effective dielectric constant of the guide.

From the above equation, it is clear that the prior knowledge of the behaviour of the dispersion characteristics of the guide (as velocity is function of the effective dielectric constant) can be used to minimize reflections over a desired frequency spectrum.

The absorbing boundary operators proposed by Keys and Higdon [5], [6] are

$$\prod_{i=1}^{p}(\frac{\partial E}{\partial x}+\frac{\cos\theta_i}{c}\frac{\partial E}{\partial t}) = 0 \qquad (5)$$

The above operator provides perfect absorption for any linear combination of plane waves traveling at incidence angles $\pm\theta_1$, $\pm\theta_2$, ... $\pm\theta_p$. It can be discretized to express the value of the field on a boundary node in terms of those on the inner neighbours.

Another boundary condition, widely used in FD-TD analysis, and simple to apply, is

$$E^k(n_x, j, k) = E^{k-n\Delta t}(n_x\text{-}1, j, k) \qquad (6)$$

where $n = \frac{\Delta l}{\Delta t}\frac{1}{v_i} = 2\sqrt{\varepsilon_{eff}(f_i)}$

3.0 Application to TLM Analysis

In TLM we deal with the scattered voltage impulses rather than directly with the electric and magnetic fields (as in the FD-TD method). The voltage impulses incident on the absorbing boundary planes are functions of both tangential (to the boundary planes) electric and magnetic fields. Since both electric and magnetic fields satisfy the wave equation, the absorbing boundary operators discussed above can be applied to either of them or to linear combinations of them. Hence, the absorbing boundary operators can be applied to the TLM impulses directly. We have implemented the absorbing boundary conditions provided by the equations (4), (5), and (6) for 3-D condensed node TLM algorithm. We have checked the performance of these boundaries by applying them at the two ends of a section (about 100 Δl length) of waveguides, microstriplines and finlines. The field values are sampled along the propagation direction and Fourier transformed to get the minimum and maximum field values at each frequency. The magnitude of reflections obtained as $\frac{VSWR-1}{VSWR+1}$ are plotted.

4.0 Results

The reflections obtained for a section of empty WR28 waveguide terminated at both ends by Higdon's absorbing boundaries ($\theta_i = 45^0$) are plotted in Figure 1. The reflections are less than 1% in the whole operating bandwidth of the waveguide. No instability was noticed for empty waveguides (no stubs in the 3-D TLM condensed node). For dielectric filled waveguides ($\varepsilon_r > 5$), instability sets in after about 1000 iterations. However, by choosing Gaussian pulse initial excitation instead of impulse excitation, the onset of instability can be significantly delayed. For microstrip and fin lines, the absorbing boundaries defined by equation (4) gave better results than others. Figure 2 shows the reflections for a section of shielded microstrip line computed with absorbing boundaries using equation (4). The values of the effective dielectric constants corresponding to absorption at two incident angles were 7.0 and 7.8. The Gaussian pulse width was 256 Δt. Figure 3 shows the reflections obtained for a section of Bilateral Finline. The effective dielectric constant values used were 0.9 and 1.5. The return loss of less than - 33 dB has been obtained over the operating bandwidth of the dominant mode (HE_1). Figure 4 shows the dispersion characteristics obtained for the same structure. The values agree well

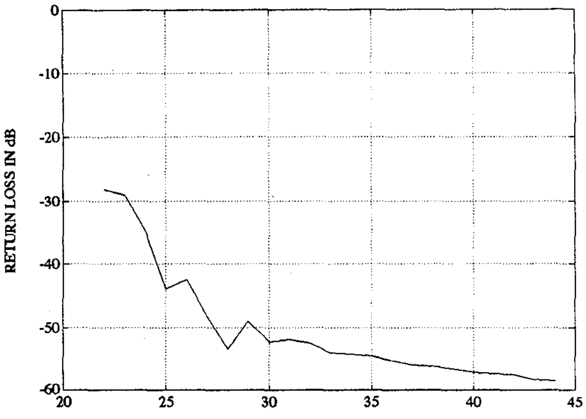

Figure 1: Reflection characteristics of back-to-back waveguide (WR28) absorbing boundaries computed with Equation 5.

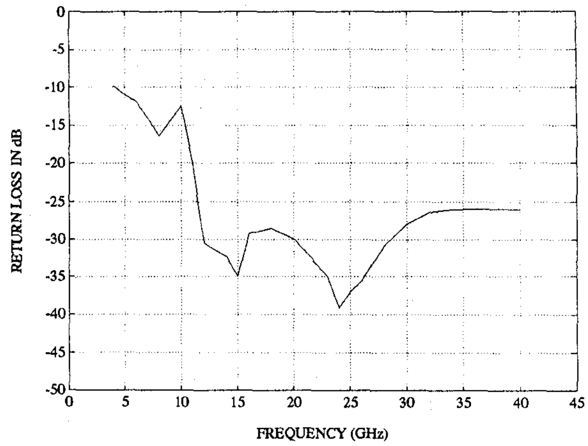

Figure 2: Reflection characteristics of back-to-back shielded microstrip absorbing boundaries computed using Equation 4 (strip width = 0. 72 mm, substrate thickness = 0. 96 mm, dielectric constant of the substrate = 9.0, shield width = 6.6 mm, shield height = 1.8 mm).

with those computed using the Spectral Domain method. Figures 5 and 6 show the reflections and dispersion characteristics for open microstrip line. In case of the open microstrip structure, the above boundary conditions do not work well for the side and top planes. This can be explained by the fact that there is no plane wave propagation at planes close to the structure (there will be reactive energy stored in the form of surface wave and radiation modes). Instead, making zero impulse reflections at these planes gave better results. As seen in the Figure 6, the return loss obtained is less than -37 dB from 0 to 20 GHz.

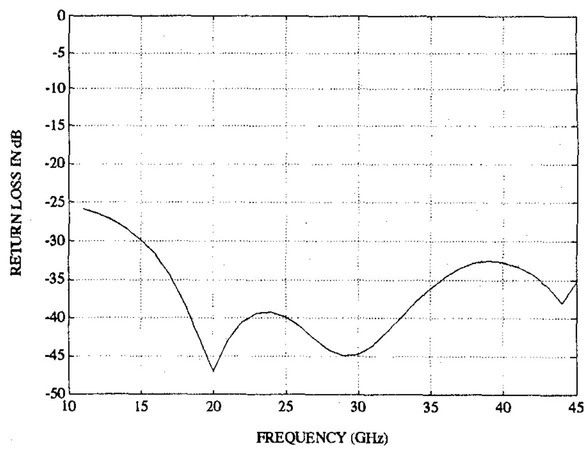

Figure 3. Reflection characteristics of back-to-back Bilateral finline absorbing boundaries computed using equation 4 (width of the waveguide housing = 7.125 mm, height of the waveguide housing = 3.5 mm, substrate thickness = 0. 5 mm, gap width = 0. 5 mm, dielectric constant of the substrate = 3.0).

5.0 Conclusions

Absorbing boundary conditions based on one-way equation have been studied and adapted for 3-D TLM analysis of planar and quasi-planar guiding structures. Reflections less than 2% over a large frequency spectrum have been obtained for microstrip and finlines. Hence, these absorbing boundaries can be applied to compute the scattering parameters of 3-D discontinuities with moderate computer resources.

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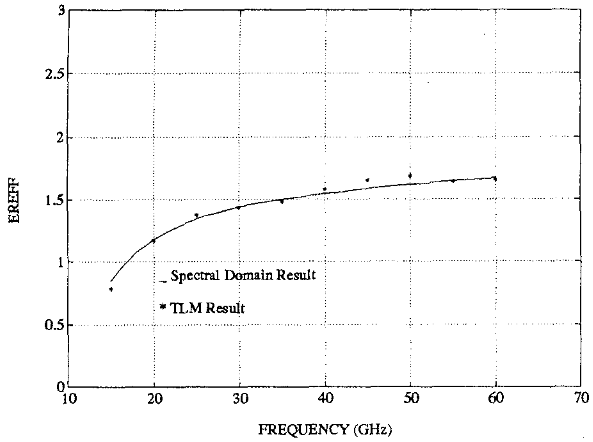

Figure 4: Dispersion characteristics of Bilateral Finline.

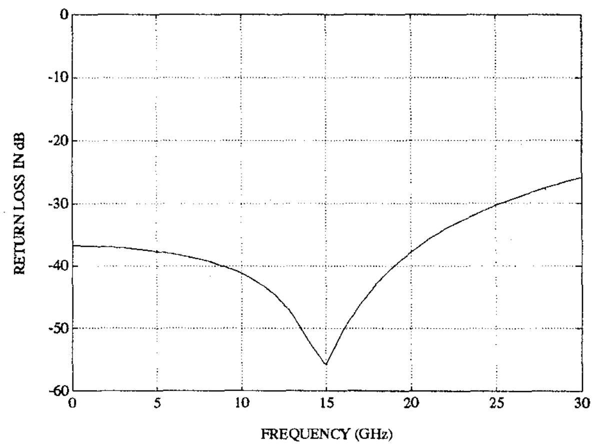

Figure 5: Reflection characteristics of back-to-back open microstrip absorbing boundaries computed using Equation 4 (width of the strip = 0. 508 mm, thickness of the substrate = 0. 635 mm, dielectric constant = 10.2.

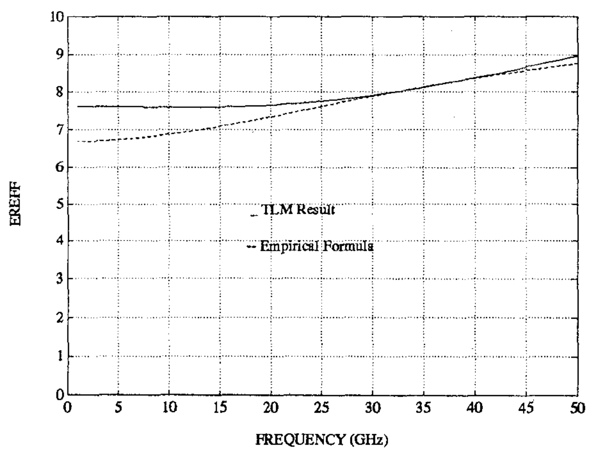

Figure 6: Dispersion characteristics of a open microstrip line.

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